Divide and Conquer: Towards Faster Pseudo-Boolean Solving

Jan Elffers

KTH Royal Institute of Technology

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Introduction

The Boolean satisfiability (SAT) problem:

> *Given Boolean variables* $x_1, \ldots, x_n$ *and set of clauses* $C_1, \ldots, C_m$*, is there assignment to the variables satisfying all clauses?*

Example:

$$(x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_2 \vee x_3)$$

Clauses are disjunctions of *literals* x, $\overline{x}$.

Introduction

Encoding as SAT used to solve various problems:

- Planning and scheduling problems.
- Hardware verification problems.
- Problems in combinatorics.

Much progress on so-called *SAT solvers* in past decades [BS97, MS99, MMZ+01].
Main algorithm: CDCL (Conflict Driven Clause Learning)

The pseudo-Boolean SAT problem

Limitation of propositional SAT:
Clauses are fairly bad at encoding real-world constraints.

We consider the generalization of SAT to *linear inequalities*.
$(x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_2 \vee x_3)$ is equivalent to $x_1 + x_2 + x_3 \geq 2$.

The pseudo-Boolean SAT problem

We represent linear inequalities over $\{0, 1\}$ in normalized form:

- All inequalities are of type $\geq$.
- Negative coefficients replaced by negative literals.

$x_1 + x_2 + x_3 \leq 1$ becomes $\overline{x}_1 + \overline{x}_2 + \overline{x}_3 \geq 2$.

We call the right hand side the *degree*.

We use c for coefficients and ℓ for literals.

Cutting planes proof system

Given a set of linear inequalities including $x_i \geq 0$, $\overline{x}_i \geq 0$ $\forall i$.
Rules:

- Addition:

$$\frac{\sum c_i \ell_i \geq w \qquad \sum c'_i \ell'_i \geq w'}{\sum c_i \ell_i + \sum c'_i \ell'_i \geq w + w'}$$

- Multiplication: for all positive integers d,

$$\frac{\sum c_i \ell_i \geq w}{\sum d \cdot c_i \ell_i \geq d \cdot w}$$

- Division: for all positive integers d,

$$\frac{\sum c_i \ell_i \geq w}{\sum \lceil c_i / d \rceil \ell_i \geq \lceil w/d \rceil}$$

Exponentially stronger than proof system underlying CDCL.

Earlier pseudo-Boolean SAT solvers

Conversion to clauses ("resolution-based"):

- MiniSat+ [ES06]
- Sat4j [LP10]
- OpenWBO [MML14]
- NaPS [SN15]

Reasoning with linear inequalities ("cutting planes-based"):

- Galena [CK05]
- Pueblo [SS06]
- Sat4j [LP10]

Our pseudo-Boolean SAT solver

We present a new pseudo-Boolean SAT solver, *RoundingSat*.
Strengths:

- Reasons with linear inequalities, so more formulas solvable.
- Highly optimized, written in C++.

The CDCL algorithm

Backtracking search, enhanced with

- Unit propagation.
- Clause learning.

The CDCL algorithm: unit propagation

If all but one literals in a clause falsified:

$$x_1 \vee x_2 \vee x_3 \vee x_4$$

then last literal must be satisfied:

$$x_1 \vee x_2 \vee x_3 \vee x_4$$

Unit propagation uses this rule to find implications.
If C propagates ℓ, then C is the *reason* of ℓ.

The CDCL algorithm: clause learning

If unit propagation falsifies a clause, derive a *learnt clause*.

Learnt clause directs search away from the conflicting state.

PB extension of CDCL

Early developments: [DG02, CK05].

- Extend unit propagation.
- Extend clause learning to pseudo-Boolean learning.

PB extension of CDCL: unit propagation

One uses *slack* function:
for $C = \sum c_i \ell_i \geq w$, ρ partial assignment,

$$slack(C, \rho) = \sum_{\ell_i \text{ not falsified by } \rho} c_i - w$$

Lower slack $\Rightarrow$ closer to propagating.

PB extension of CDCL: learning

We use *generalized resolution* to combine linear inequalities.

Takes linear combination such that some variable occuring with opposite signs cancels.

$$
\begin{aligned}
& Res(2x + y \geq 1, \overline{x} + \overline{z} \geq 1, x) \\
= {} & Res(2x + y \geq 1, 2\overline{x} + 2\overline{z} \geq 2, x) \\
= {} & 2x + y + 2\overline{x} + 2\overline{z} \geq 1 + 2 \\
= {} & y + 2\overline{z} \geq 1
\end{aligned}
$$

PB extension of CDCL: execution example

Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6.$
- $C' : 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3.$

PB extension of CDCL: execution example

Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6.$
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We set $x_1 = 0$.

PB extension of CDCL: execution example

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We set $x_1 = 0$.
C propagates x_2, x_3 and x_4.

PB extension of CDCL: execution example

Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.
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We set $x_1 = 0$.
C propagates x_2, x_3 and x_4.
Now C' is falsified, so we start conflict analysis.

PB extension of CDCL: execution example

Given two constraints

- $C : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6.$
- $C' : 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3.$

We set $x_1 = 0$.
C propagates x_2, x_3 and x_4.
Now C' is falsified, so we start conflict analysis.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\mathrm{confl}} = C'$.
- $\mathit{reason}(x_2, \rho) = \mathit{reason}(x_3, \rho) = \mathit{reason}(x_4, \rho) = C$.

PB extension of CDCL: learning

while *termination criterion does not hold* **do**
  $\ell \leftarrow$ literal assigned last on the trail ρ;
  if $\overline{\ell}$ *occurs in* C_{confl} **then**
    $C_{\mathrm{reason}} \leftarrow \mathit{reason}(\ell, \rho)$;
    $C_{\mathrm{reason}} \leftarrow \mathit{reduceReason}(C_{\mathrm{reason}}, C_{\mathrm{confl}}, \ell, \rho)$;
    $C_{\mathrm{confl}} \leftarrow \mathit{Res}(C_{\mathrm{confl}}, C_{\mathrm{reason}}, \overline{\ell})$;
  end
  $\rho \leftarrow \mathit{removeLast}(\rho)$;
end
return C_{confl};

(Green: new compared to CDCL)

PB extension of CDCL: reason reduction

We discuss the method of [CK05] and the one of RoundingSat.

Operations used:

- *Weakening*: if $x_1 + x_2 + \mathbf{x_3} \geq 2$, then $x_1 + x_2 \geq 1$.
- *Saturation*: if $x + \mathbf{3}y \geq 2$, then $x + \mathbf{2}y \geq 2$.
- *Division*: as defined before,

$$\frac{\sum c_i \ell_i \geq w}{\sum \lceil c_i/d \rceil \ell_i \geq \lceil w/d \rceil}$$

Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$$

1.
2.

Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\mathrm{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\mathrm{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$$

1. **Try generalized resolution.**
2.

Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$$

$$\mathit{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : x_5 \geq 1$$

1. **Try generalized resolution.**
2.

Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
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$$C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$$

$$\mathit{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : x_5 \geq 1$$

1. Try generalized resolution.
2. **If not falsified, weaken non-falsified literal and saturate.**

Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 \qquad + 2x_3 + 2x_4 + x_5 \geq 4$$

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Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\mathrm{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\mathrm{reason}} : 2x_1 \qquad\qquad + 2x_3 + 2x_4 + x_5 \geq 4$$

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Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $reason(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 \qquad\qquad + 2x_3 + 2x_4 + x_5 \geq 4$$

$$Res(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : 2\overline{x}_2 + x_5 \geq 1$$

1. **Try generalized resolution.**
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Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 \qquad\quad + 2x_3 + 2x_4 + x_5 \geq 4$$

$$\mathit{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : 2\overline{x}_2 + x_5 \geq 1$$

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$$C_{\text{reason}} : 2x_1 \qquad\qquad\qquad + 2x_4 + x_5 \geq 2$$

1. Try generalized resolution.
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Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
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$$C_{\mathrm{reason}} : 2x_1 \qquad\qquad + 2x_4 + x_5 \geq 2$$

$$Res(C_{\mathrm{confl}}, C_{\mathrm{reason}}, \overline{x}_4) : 2\overline{x}_2 + 2\overline{x}_3 + x_5 \geq 1$$

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$$C_{\text{reason}} : 2x_1 \qquad\qquad + 2x_4 + x_5 \geq 2$$

$$\mathit{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : 2\overline{x}_2 + 2\overline{x}_3 + x_5 \geq 1$$

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$$C_{\mathrm{reason}} : 2x_1 \qquad\qquad\qquad + 2x_4 \qquad \geq 1$$

1. Try generalized resolution.
2. **If not falsified, weaken non-falsified literal and saturate.**

Reason reduction of [CK05]

Reason reduction example.

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
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$$C_{\text{reason}} : \quad x_1 \qquad\qquad\qquad + \quad x_4 \qquad \geq 1$$

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$$C_{\text{reason}} : \quad x_1 \qquad\qquad\qquad + \quad x_4 \qquad \geq 1$$

1. **Try generalized resolution.**
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Reason reduction example.

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- $C_{\mathrm{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\mathrm{reason}} : \quad x_1 \qquad\qquad\qquad + \quad x_4 \qquad \geq 1$$

$$\mathit{Res}(C_{\mathrm{confl}}, C_{\mathrm{reason}}, \overline{x}_4) : 2\overline{x}_2 + 2\overline{x}_3 \geq 1$$

1. **Try generalized resolution.**
2. If not falsified, weaken non-falsified literal and saturate.

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- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
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- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : \quad x_1 \qquad\qquad\qquad + \quad x_4 \qquad \geq 1$$

$$\mathit{Res}(C_{\text{confl}}, C_{\text{reason}}, \overline{x}_4) : 2\overline{x}_2 + 2\overline{x}_3 \geq 1$$

1. **Try generalized resolution.** Works, so terminate.
2. If not falsified, weaken non-falsified literal and saturate.

Reason reduction of RoundingSat

Same example:

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\text{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\text{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$$

Reason reduction of RoundingSat

Same example:

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\mathrm{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, *reason*$(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\mathrm{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$$

1. Weaken non-falsified literals in C_{reason} with coefficient not divisible by coefficient of x_4.

Reason reduction of RoundingSat

Same example:

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\mathrm{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\mathrm{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 \qquad \geq 5$$

1. Weaken non-falsified literals in C_{reason} with coefficient not divisible by coefficient of x_4.

Reason reduction of RoundingSat

Same example:

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\mathrm{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, $\mathit{reason}(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\mathrm{reason}} : 2x_1 + 2x_2 + 2x_3 + 2x_4 \qquad \geq 5$$

1. Weaken non-falsified literals in C_{reason} with coefficient not divisible by coefficient of x_4.
2. Divide by coefficient of x_4.

Reason reduction of RoundingSat

Same example:

- $\rho = (\overline{x}_1, x_2, x_3, x_4)$.
- $C_{\mathrm{confl}} = 2\overline{x}_1 + 2\overline{x}_2 + 2\overline{x}_3 + 2\overline{x}_4 \geq 3$.
- $\ell = x_4$, *reason*$(\ell, \rho) = 2x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 \geq 6$.

$$C_{\mathrm{reason}} : \; x_1 + \; x_2 + \; x_3 + \; x_4 \qquad \geq 3$$

1. Weaken non-falsified literals in C_{reason} with coefficient not divisible by coefficient of x_4.
2. Divide by coefficient of x_4.

Experimental results: PB16 decision track, small integers

Entries: number of solved instances (satisfiable + unsatisfiable)

Bold: solver is (one of) the best in category

	RoundingSat	*Sat4j Res+CP*	*Sat4j Res*	*Open-WBO*
PB05 aloul	**36 + 21**	**36 + 21**	36 + 3	36 + 6
PB06 manquiho	**14 + 0**	**14 + 0**	**14 + 0**	3 + 0
PB06 ppp-problems	4 + 0	4 + 0	4 + 0	3 + 0
PB06 uclid	1 + 47	1 + 47	1 + 47	1 + 49
PB06 liu	16 + 0	16 + 0	16 + 0	17 + 0
PB06 namasivayam	72 + 128	72 + 128	72 + 128	72 + 128
PB06 prestwich	10 + 0	11 + 0	9 + 0	**14 + 0**
PB06 roussel	**0 + 22**	**0 + 22**	0 + 4	0 + 4
PB10 oliveras	34 + 32	34 + 32	34 + 33	34 + 33
PB11 heinz	2 + 0	2 + 0	2 + 0	2 + 0
PB11 lopes	42 + 26	37 + 25	37 + 25	33 + 28
PB12 sroussel	**31 + 0**	21 + 0	23 + 0	**29 + 1**
PB16 elffers	**0 + 287**	0 + 229	0 + 142	0 + 213
PB16 nossum	**68 + 0**	39 + 0	39 + 0	55 + 0
PB16 quimper	43 + 214	43 + 213	43 + 213	**46 + 241**
Sum	**373 + 777**	330 + 717	330 + 595	345 + 703

Experimental results

- RoundingSat dominates Sat4j (both versions).
- RoundingSat and Sat4j Res+CP better than resolution-based solvers on 3 categories.
- OpenWBO sometimes better than RoundingSat, sometimes worse.

Conclusion

RoundingSat shows that reasoning with linear inequalities can be competitive on many different domains.
And sometimes, it is crucial for performance.

Future work:

- Extend to optimization track in non-trivial way.

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And sometimes, it is crucial for performance.

Future work:

- Extend to optimization track in non-trivial way.

Thank you!

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